

A Simple Absorption Model for High-Energy π - N , K - N , and N - N Scattering: Application to Charge-Exchange Reactions†

E. M. HENLEY AND I. J. MUZINICH

University of Washington, Seattle, Washington

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A simple parametrization of the high-energy π - N , K - N , and N - N scattering phase shifts is obtained. The partial-wave amplitudes are assumed to be purely imaginary. This representation is used to discuss the absorption in a peripheral-model description of charge-exchange reactions with incident pions, kaons, and nucleons. The Jacob-Wick helicity amplitudes are used to treat the spin dependence. The n - p charge-exchange differential cross section is found to be narrow, primarily because, in the presence of absorption, the pion-exchange contribution is not zero in the forward direction. In addition the angular distribution exhibits a secondary maximum at a small angle due to one of the helicity-flip amplitudes. In contrast, the π - N and K - N charge-exchange angular distributions, determined by ρ exchange, are relatively wide.

I. INTRODUCTION

WE have found a simple parametrization for high-energy nucleon-nucleon, kaon-nucleon, and pion-nucleon scattering. The simple model is able to reproduce approximately the following features: (a) the total cross section,¹ (b) the ratio of the elastic to the total cross section,¹ (c) the small-angle diffraction scattering (but without shrinking),^{1,2} (d) the large-angle elastic scattering.^{2,3} In addition, our scattering amplitude has some of the expected analytic properties anticipated at high energies and is particularly suitable for carrying out distorted wave calculations for peripheral processes.^{4,5} We apply it here to n - p charge-exchange scattering⁶ and use it to predict the differential cross section for the pion-nucleon and kaon-nucleon charge-exchange reactions. The narrowness of the former is due to the pion exchange, which does not contribute to the latter reaction. Consequently the π - p and K - p charge-exchange angular distributions are predicted to be even somewhat wider than the elastic differential cross section.

II. PARAMETRIZATION

The parametrization that we suggest at high energies invokes purely imaginary phase shifts, and is

$$\chi_l \equiv 1 - e^{2i\delta_l} = \frac{C \exp\{RA^{-1}[1 - (1 + l^2/k^2 R^2)^{1/2}]\}}{(1 + l^2/k^2 R^2)^{1/2}}, \quad (1)$$

where $C \leq 1$, A , R are three adjustable parameters. However, except for the large-angle elastic scattering, the results are primarily dependent on C and AR . The

† Supported in part by the U. S. Atomic Energy Commission under contract A. T. (45-1)-1388.

¹ See, for example, K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **11**, 425 and 503 (1963).

² J. Orear, Phys. Rev. Letters **12**, 112 (1964).

³ See, for instance, R. Serber, Phys. Rev. Letters **10**, 357 (1963).

⁴ L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters **12**, 399 (1964). A. Dar and W. Tobocman, *ibid.* **12**, 511 (1964).

⁵ M. Ross and G. L. Shaw, Phys. Rev. Letters **12**, 627 (1964).

⁶ H. Palevsky, J. A. Moore, R. L. Stearns, H. R. Muether, R. J. Sutter, R. E. Chrien, A. P. Jain, and K. Otnes, Phys. Rev. Letters **9**, 509 (1962).

critical value of l for which $\chi_l/\chi_0 = e^{-1}$ is given by l_c ,

$$l_c^2 = k^2[(R+A)^2 - R^2] \approx 2ARk^2, \quad (2)$$

where the latter equality holds if $R/A \gg 1$.

In terms of the above constants, the differential elastic cross section is given by³

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= |f(\theta)|^2 \\ &\approx |ik \int_0^\infty J_0(k_t \rho) \chi(\rho) \rho d\rho|^2 \\ &= (kARC)^2 \frac{\exp\{-2RA^{-1}[(1+k_t^2 A^2) - 1]\}}{1+k_t^2 A^2} \end{aligned} \quad (3a)$$

$$\approx (kARC)^2 e^{-k_t^2 AR} \quad \text{for } k_t A \ll 1 \quad (3b)$$

$$\approx (kRC/k_t)^2 e^{-2k_t R} \quad \text{for } k_t A \gg 1. \quad (3c)$$

The appearance of $k_t \equiv k \sin\theta$ occurs from the asymptotic replacement of $P_l(\cos\theta)$ by $J_0(k_t \rho)$. It has been argued⁷ that for large angles $2k \sin^2\theta$ should be substituted for k_t , but this is not done below. The replacement would not affect our detailed comparisons with experiments, which are restricted to angles less than 40° . The parameter ρ corresponds to l/k . The elastic and total cross sections are³

$$\begin{aligned} \sigma_{el} &= 2\pi \int_0^\infty \chi^2(\rho) \rho d\rho \\ &= -2\pi(RC)^2 e^{R/A} \text{Ei}(-2R/A) \approx \pi ARC^2, \end{aligned} \quad (4)$$

$$\sigma_{total} = 4\pi \int_0^\infty \chi(\rho) \rho d\rho = 4\pi ARC, \quad (5)$$

where the function Ei is the exponential integral.⁸ We note that our model contains both the small-angle

⁷ R. J. Glauber, in *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1959), p. 345. R. Serber, Rev. Mod. Phys. **36**, 649 (1964); and E. M. Henley and D. U. L. Yu, Phys. Rev. **135**, B1152 (1964).

⁸ E. Jahnke and M. Ende, *Tables of Functions* (Dover Publications, Inc., New York, 1943).

diffraction behavior [$d\sigma/d\Omega \sim \exp(ARt)$, with t the invariant four-momentum transfer] and the larger angle exponential behavior suggested by Orear.² Furthermore, except for the large-angle data, only the constants AR and C enter. The ratio of the elastic to the total cross section is $0.25C$. For N - N scattering (N stands for neutron or proton), the data¹ suggest $C=1$. On the other hand, for π - p and K - p scattering this ratio is ≈ 0.18 , so that $C \approx 0.72$. The total cross section for both p - p (≈ 40 mb) and π - p (≈ 28 mb) collisions then determines $AR \approx 0.32$ F². This parameter also describes the measured small-angle elastic scattering. For instance, Orear² uses $AR \approx 0.36$ F², but with a form which corresponds to Eq. (3b) rather than (3a); the latter requires a somewhat smaller value of AR , in agreement with our choice. The additional parameter R is at our disposal to fit the large-angle scattering, Eq. (3c). The value suggested by Orear is $R \approx \frac{2}{3}$ F; because of our denominator, a somewhat smaller value of R gives a better fit and in the calculations below we use $R \approx 0.6$ F.

In addition to the above features, the model gives a branch point for the scattering amplitude at $t \approx k_i^2 = A^{-2} \approx 7m_\pi^2$. Analytic properties dictate that the threshold for the cut occur for $t = 4m_\pi^2$, so that our choice of constants is not inconsistent with this feature.

Because of the shrinking of the p - p diffraction pattern with increasing energy,¹ the above parameters are not too useful for lower energy nucleon-nucleon processes (say, below 5 BeV). For instance, at 2.85 BeV, which corresponds to the highest energy at which the n - p charge exchange reaction has been measured,⁶ the observed angular distribution for elastic p - p scattering is considerably wider than given by the above form; furthermore, the ratio of the elastic to the total cross section is 0.36 rather than 0.25. These characteristics imply that χ_i may not be purely real and that the transition from no absorption ($\chi_i=0$) to full absorption ($\chi_i=1$) given by Eq. (1) is too gradual. For a sharp transition, with $C=1$,

$$\begin{aligned} \chi_i &= 1 & \text{for } l < l_c, \\ \chi_i &= 0 & \text{for } l > l_c, \end{aligned} \quad (6)$$

the ratio of elastic to total cross section is 0.5, but such a sharp boundary leads to secondary diffraction maxima in the elastic differential cross section, contrary to observation.⁹ At this energy, the situation appears to be intermediate between Eqs. (1) and (6).

III. CHARGE EXCHANGE

A. $n+p \rightarrow p+n$

The above considerations are applied to the n - p , K - p , and π - p charge exchange reactions. Of the known

particles and resonances only the π and ρ can be exchanged¹⁰ in $n+p \rightarrow p+n$; and only the ρ in $\pi^-+p \rightarrow \pi^0+n$, and $K^-+p \rightarrow \bar{K}^0+n$. Furthermore, in perturbation theory (or peripheral model) of $n+p \rightarrow p+n$ only the ρ contributes at very small angles; the pion effect is zero in the forward direction. However, in the distorted-wave Born approximation the pion not only contributes,¹¹ but dominates the angular distribution at forward angles, as we shall show.

With the use of the WKB approximation¹² the distorted-wave Born amplitudes for nucleon-nucleon scattering can be written as¹³

$$\begin{aligned} E\phi_{\lambda'\lambda\mu'\mu} &= \sum_J (2J+1)(1-\chi_J) B_{\lambda'\lambda\mu'\mu}^J d_{\lambda'-\lambda, \mu'-\mu}^J(\theta) \\ &= E\phi_{\lambda'\lambda\mu'\mu}^B - \sum_J (2J+1)\chi_J B_{\lambda'\lambda\mu'\mu}^J d_{\lambda'-\lambda, \mu'-\mu}^J(\theta), \end{aligned} \quad (7)$$

where $B_{\lambda'\lambda\mu'\mu}^J$ is the partial-wave helicity amplitude¹⁴ for the single particle exchanged and $\phi_{\lambda'\lambda\mu'\mu}^B$ is the corresponding single-particle-exchange helicity amplitude. The parameter χ_J is given by Eq. (1); $d_{\lambda'-\lambda, \mu'-\mu}^J$ are well-known rotation matrices¹⁴; E is the energy of either particle in the c.m. frame; λ', λ are the helicities of the final nucleons and μ', μ the corresponding ones for the initial nucleons. The pion exchange, which contributes to the helicity amplitudes $\phi_{+ + - -}$ and $\phi_{+ - - +}$, and the ρ exchange, which contributes to all five independent helicity amplitudes, do not interfere in the cross section.¹⁰ With the factor $(1-\chi_J)$, which cuts off the lower angular momenta, $\phi_{+ + - -}$ no longer vanishes for the pion in the forward direction; however, $\phi_{+ - - +}$ is still zero for the π , since it is expanded in terms of functions $d_{1, -1}^J(\theta)$ which are zero at $\theta=0^\circ$.

Unfortunately, the highest energy measurements reported to date⁶ have been carried out at 2.85 BeV, where the parameterization given by Eq. (1) is no longer very good. Nevertheless, we have carried out the partial-wave sum in Eq. (7) with this form. With the insertion of the proper isospin factors, we compare our result with the data in Fig. 1. The secondary maximum at $\theta=15^\circ$ is caused by the amplitude $\phi_{+ - - +}$ reaching a maximum at this angle. In the same figure, we also show the differential cross section obtained with only pion exchange, as well as with pion exchange in the absence of the amplitude $\phi_{+ - - +}$. It is clear from the figure that the pion contribution dominates the cross section at small angles. With values of $g_{\rho NN}^2/4\pi \approx 2$ and $g_{\pi NN}^2/4\pi \approx 14$, we find a differential cross section of 3.2 mb/sr at 0° in the c.m. system. This compares favorably with the experimental cross section of 3.0 ± 0.5 mb/sr. Although there is a small imaginary part of the amplitude that has been neglected, its contribution is less than 0.3 mb/sr at 0° for a 4-mb difference between the pp and np total cross sections. Thus, the imaginary part

¹⁰ See, for instance, I. J. Muzinich, Phys. Rev. Letters **11**, 88 (1963); M. M. Islam and T. W. Preist, *ibid.* **11**, 444 (1963).

¹¹ D. V. Bugg, Phys. Letters **7**, 365 (1963).

¹² See, e.g., L. I. Schiff, Phys. Rev. **103**, 443 (1956).

¹³ K. Gottfried and J. D. Jackson (to be published).

¹⁴ M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

⁹ T. Fujii, G. B. Chadwick, G. B. Collins, P. J. Duke, N. C. Hien, M. A. R. Kemp, and F. Turkot, Phys. Rev. **128**, 1836 (1962).

of the amplitude does not spoil the agreement between our model and experiment. In fact, for a sharp cutoff, Eq. (6), with $l_c = kR_c \approx 6$ the charge-exchange angular distribution is even somewhat sharper than with Eq. (1) and the magnitude of the cross section is reduced to about 2 mb/sr. Thus, it is clear that one can understand both the sharp angular distribution as well as the magnitude of the charge-exchange cross section on the basis of a simple peripheral model with absorption.

B. $\pi^- + p \rightarrow \pi^0 + n$

The above calculation has been repeated for the reaction $\pi^- + p \rightarrow \pi^0 + n$ at the same c.m. momentum, corresponding to a laboratory kinetic energy of 3.15 BeV as well as at 6 BeV. Unlike the case of np charge exchange, only the ρ contributes, and we find a considerably wider angular distribution. The experimental π^-p elastic scattering does not show any shrinking and Eq. (1) (with the chosen parameters) gives a reasonable fit to the total cross section and elastic-scattering data¹⁵ even at 3.15 BeV as shown in Fig. 2.

For the charge-exchange reaction the distorted-wave helicity amplitudes in the c.m. are^{14,16}

$$W f_{\lambda,\mu} = W f_{\lambda,\mu}^B - \sum_J (2J+1) \chi_J B_{\lambda,\mu}^J d_{\lambda,\mu}^J(\theta), \quad (8)$$

where W is the total energy, $f_{\lambda,\mu}^B$ is the single-particle exchange (Born term) helicity amplitude, and $B_{\lambda,\mu}^J$ is given by ($\hbar = c = 1$)

$$B_{\lambda,\pm\lambda}^J = \frac{g_{\rho\pi\pi} g_{\rho NN}}{4\pi} \left(\frac{1 + E\omega/k^2}{M\omega/k^2} \right) \times [Q_{J+1/2}(\xi) \pm Q_{J-1/2}(\xi)]. \quad (9)$$

In Eq. (9), E is the energy of the nucleon, ω is that of the pion, k is the momentum of either particle, and $\xi = 1 + m_\rho^2/2k^2$, where m_ρ is the mass of the ρ meson.

In Fig. 2 we exhibit the relative charge exchange differential cross section, at 3.15 BeV arbitrarily normalized at 0° . The angular distribution out to 40° is only slightly narrower for a sharp cutoff ($l_c = kR_c$, $R_c \sim 1$ F, and $C = 0.72$), Eq. (6), than for our representation, Eq. (1) which is plotted. The differential cross section at 0° is 3.7 mb/sr with Eq. (1) and 1.3 mb/sr with Eq. (6), if we use the generally accepted values¹⁷ of $g_{\rho\pi\pi}^2/4\pi \approx g_{\rho NN}^2/4\pi \approx 2$. Although the relatively broad angular distribution is in agreement with preliminary results obtained at 4 BeV by Faissner, Ferrero, Gerber, Reinharz, and Stein,¹⁸ the magnitude of the 0° cross section is too large, even with a sharp cut-off. It thus appears that a smaller coupling constant

¹⁵ C. C. Ting, L. W. Jones, and M. L. Perl, Phys. Rev. Letters **9**, 468 (1962).

¹⁶ G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

¹⁷ J. J. Sakurai, in Proceedings of the International School of Physics, Varenna, Italy, 1962 (to be published).

¹⁸ H. Faissner, F. Ferrero, H. J. Gerber, M. Reinharz, and J. Stein (private communication).

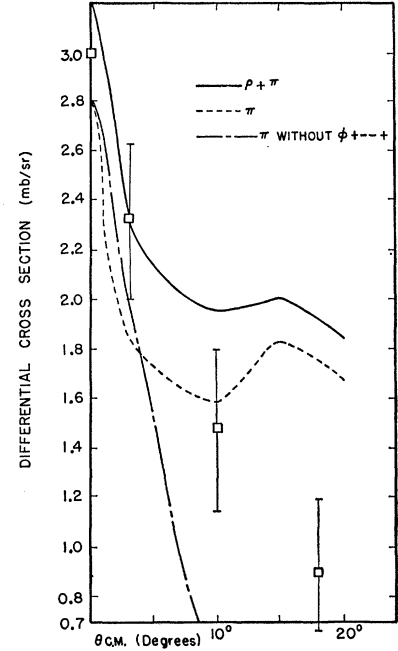


FIG. 1. Charge exchange scattering at 2.85 BeV. The experimental points are taken from Ref. 6. The solid curve corresponds to the proposed high-energy model with the inclusion of both ρ and π contributions. The dashed curves show the pion contribution; that with the long and short dashes omits the helicity amplitude ϕ_{+---+} .

or C closer to 1 (rather than 0.72) is required to fit the magnitude of the π^-p charge-exchange cross section. The larger C is also in agreement with a larger ratio $\sigma_{el}/\sigma_{tot} (\approx 0.22)$ at this energy. Thus, with $C = 0.8$, the cross section is 0.7 mb/sr at 0° with the sharp cutoff. However, the choice of 3.15 BeV is not a particularly suitable one to test our model, since recent measure-

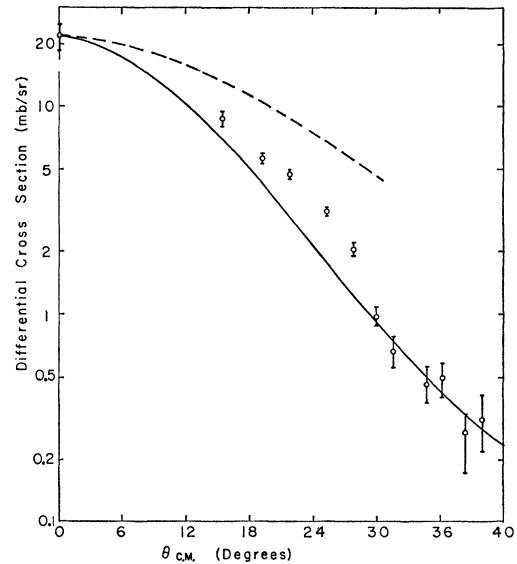


FIG. 2. Differential cross sections for π^-p scattering at 3.15 BeV. The experimental points are those for elastic scattering measured by Ting, Jones, and Perl (Ref. 15). The solid curve is that calculated for $\pi^-p \rightarrow \pi^-p$, normalized at 0° by a factor of 1.2 and the dashed one is that for $\pi^-p \rightarrow \pi^0n$, arbitrarily normalized at 0° .

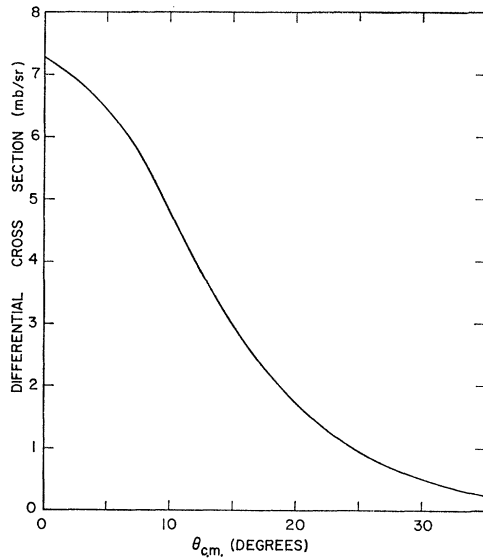


FIG. 3. Differential cross section for the reactions $\pi^- + p \rightarrow \pi^0 + n$ 6 BeV and $K^- + p \rightarrow \bar{K}^0 + n$ at 5.8 BeV, based on Eq. (1). In both cases $k=11.7m_\pi$. The angular dependence is the same for both reactions, but the ordinate for the K^-p charge-exchange cross section should be multiplied by 0.5.

ments¹⁹ show a resonance and considerable structure close to this energy. We have repeated our calculation at 6 BeV, where the parametrization given by Eq. (1) is expected to be better. The differential charge exchange cross section for this energy is plotted in Fig. 3.

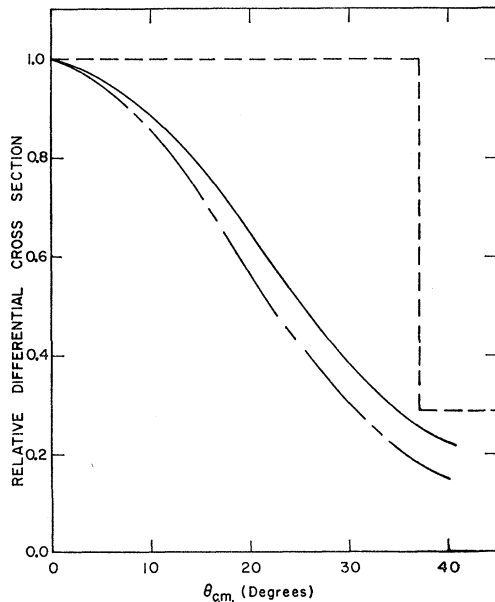


FIG. 4. Relative differential cross section for the reaction $K^- + p \rightarrow \bar{K}^0 + n$ at 2 BeV/c. The histogram is that obtained in reference 20, the solid curve corresponds to Eq. (1) and the long and short dash one to Eq. (6) with $l_c \approx 6$.

¹⁹ M. A. Wahling, I. Mannelli, L. Sodickson, O. Fackler, C. Ward, T. Kan, and E. Shibata, Phys. Rev. Letters **13**, 103 (1964).

C. $K^- + p \rightarrow \bar{K}^0 + n$

The development of the previous section applies also to the K^-p charge-exchange reaction. In fact, Eqs. (8) and (9) are equally valid for this case, if ω is taken to be the energy of the kaon and $g_{\rho\pi\pi}$ is replaced by $g_{\rho KK}(2)^{-1/2}$ where the factor $(2)^{-1/2}$ comes from isospin considerations. SU_3 symmetry predicts that $g_{\rho KK} = g_{\rho\pi\pi}$ when defined in the above manner.¹⁷ At the present time the highest energy at which data²⁰ exist is at 2 BeV/c. In Fig. 4 we compare our calculated angular distribution to the measured one at angles $\leq 40^\circ$, where the peripheral model should be applicable. Again, for these angles the sharp and rounded cutoffs yield only slightly different angular distributions. However, the former case (with $C=0.72$, $l_c \approx kR_c$ and $R_c \approx 1$ F) gives a forward differential cross section of 0.4 mb/sr, whereas the latter one yields 1.1 mb/sr compared to a measured cross section of the order of 0.4 mb/sr. Thus the peripheral absorption model accounts well for both the magnitude and angular shape. At 3 BeV, the calculated forward cross section is 0.6 mb/sr for the sharp cutoff and 1.9 mb/sr for Eq. (1). The shape of the angular distribution is essentially identical to that for the pion charge-exchange reaction at 3.15 BeV shown in Fig. 2. In fact the ratio of $(d\sigma/d\Omega)(0^\circ)$ for $K^- + p \rightarrow \bar{K}^0 + n$ to that for $\pi^- + p \rightarrow \pi^0 + n$ can be used to test the SU_3 prediction of the coupling constants. This ratio is given by

$$R = \left| \frac{g_{\rho KK} k^2 + E(k^2 + m_K^2)^{1/2}}{g_{\rho\pi\pi} k^2 + E(k^2 + m_\pi^2)^{1/2}} \right|^2 \quad (10)$$

at the same c.m. momentum. In fact, the above ratio is approximately valid at other angles as well (if the c.m. momenta are identical) because the helicity flip amplitude is small. This conclusion is independent of the details of the absorption and depends only on the validity of the peripheral model. At 5.8 BeV, where our parametrization Eq. (1) is expected to be valid, the differential cross section is shown in Fig. 3.

IV. CONCLUSION

We conclude that the peripheral model, modified to take absorption into account, is capable of explaining many of the characteristic features of charge-exchange reactions. In particular the pion exchange in $n + p \rightarrow p + n$ accounts for the sharp fall-off at small angles. However, in the πp and Kp charge-exchange reactions the dominant peripheral contribution comes from the ρ -meson exchange and absorption, and the angular distribution is considerably wider. This is in contrast to the prediction based upon the dominance of the ρ -meson Regge trajectory.¹⁰

²⁰ D. Barge, W. Chu, L. Leipuner, R. Crittenden, H. J. Martin, F. Ayer, L. Marshall, A. C. Li, W. Kernan, and M. L. Stevenson, Phys. Rev. Letters **13**, 69 (1964).

In the $n\bar{p}$ case one of the helicity flip amplitudes, ϕ_{+--+} , causes a secondary maximum to appear at small angles ($\approx 15^\circ$). Although this maximum might be washed out, more experimental data would be valuable to study this feature of the absorption model.

The ratio of the $\pi\bar{p}$ to $K\bar{p}$ charge-exchange cross sections at the same c.m. momentum, particularly at 0° , can be used to test the SU_3 prediction of coupling constants. However, this test should be carried out at higher energies (e.g., $\gtrsim 6$ BeV) than where data presently exist in order to get outside the $\pi\bar{p}$ resonance

region. Thus, additional data, especially at higher energies would clearly be of value.

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Electromagnetic Form Factor of the Neutrino*

WEN-KWEI CHENG AND S. A. BLUDMAN

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania

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Bernstein and Lee and, independently, Meyer and Schiff have recently published calculations of the neutrino electromagnetic form factor, obtaining results differing by a finite constant term. This difference can be traced back to how the W -meson contribution is regularized: The Bernstein-Lee calculation is gauge-invariant at every step, while Meyer and Schiff simply impose over-all neutrino charge neutrality at the end. The ξ -limiting process in addition sums a class of electromagnetic radiative corrections and assigns the value $\ln\alpha^{-1}$ to the logarithmically divergent term in the W -meson contribution. Since the finite term, which is almost comparable to $\ln\alpha^{-1}$ in magnitude, is not fixed by the ξ -limiting method, the neutrino form factor has actually been determined only to order of magnitude by this method. For this reason and because the W mass is large (or infinite), we have determined the largest part of the neutrino form factor from the charged lepton contribution using a gauge-invariant direct-interaction theory. This is obtained, without further calculation, from the photon vacuum polarization. The ν_e charge radius thus measures the same integral that appears in the perturbation-theory calculation of $Z_3 e^{1/2}$, the charge renormalization in quantum electrodynamics.

INTRODUCTION

IN the weak interaction theory, either based on the local four-fermion current-current self-interaction (F theory) or on the intermediate boson model (W theory), $(e\nu_e)$ $(e\nu_e)$ and $(\mu\nu_\mu)$ $(\mu\nu_\mu)$ couplings would exist to the lowest order in the weak coupling constant G (F theory) or g^2 (W theory). An immediate consequence of this interaction is that the neutrinos would have electromagnetic interaction through the generation of a charge form factor in the sequence¹:

$$(i) \text{ } F \text{ theory: } \nu_l \rightleftharpoons \nu_l + l^+ + l^- \rightleftharpoons \nu_l + \gamma, \quad (1)$$

$$(ii) \text{ } W \text{ theory: } \nu_l \rightleftharpoons l^- + W^+ \rightleftharpoons l^- + W^+ + \gamma \rightleftharpoons \nu_l + \gamma, \quad (2)$$

where $l=e$ or μ . The matrix element of the neutrino electromagnetic current operator J_μ evaluated between initial and final one-neutrino states in a γ_5 -invariant

CP -invariant theory² is

$$\langle \nu' | J_\mu | \nu \rangle = i\bar{\nu}(p')\gamma_\mu(1+\gamma_5)\nu(p)F(q^2), \quad (3)$$

where p and p' are, respectively, the initial and final four momenta and $q^2=(p-p')^2$. $F(q^2)$ is the neutrino form factor, which, in lowest order electromagnetic and weak interaction, originates from the Feynman diagrams of Figs. 1 or 2.

The explicit form of $F(q^2)$ has recently been calculated by Bernstein and Lee³ and independently by Meyer and Schiff⁴ in the W theory for the case of vector

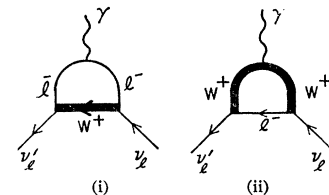


FIG. 1. The lowest order diagrams contributing to the neutrino form factor ($l=e$ or μ) in the W theory.

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¹ In order to satisfy gauge invariance or energy-momentum conservation, the coupling must be to photons off the $q^2=0$ mass shell, i.e., to virtual photons or to plasmons.

² We do not consider the question of whether a consistent quantum electrodynamics exists for a massless spinor field.

³ J. Bernstein and T. D. Lee, Phys. Rev. Letters 11, 512 (1963).

⁴ Ph. Meyer and D. Schiff, Phys. Letters 8, 217 (1964).